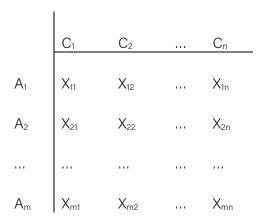
1. Construction of the decision matrix and determination of the weight of criteria. The multicriteria problem can be expressed in matrix format in the following way:



where:

A<sub>1</sub>, A<sub>2</sub>,..., A<sub>m</sub> – alternative of internal insulation systems/materials

 $C_1$ ,  $C_2$ , ...  $C_n$  - the criteria for which the alternative performance is measured

x - the value of alternative  $A_i$  with respect to the criterion  $C_j$ .

Criteria of the functions can be either benefit functions when more is better, e.g. wall surface temerpature,  $CO_2$  reduction, reduction of energy and other costs, reduction of energy consumption or loss functions when less is better, e.g. total costs of internal insulation. The relative importance of each criterion is given by a set of weights which are normalized to sum to one.

Let X = (xij) be a decision matrix and  $W = [w_1, w_2, ..., w_n]$  a weight vector, where  $xij \in \Re$ ,  $wj \in \Re$  and w1 + w2 + ... + wn = 1.

2. Calculation of the normalized decision matrix.

Various attribute dimensions are transformed into non-dimensional attributes. It allows comparisons across criteria. Since various criteria are measured in various units, the scores in the evaluation matrix X have to be normalised to one scale. The normalization of values is carried out by standardized formula and the normalized value n<sub>ij</sub> is calculated as:

$$n_{ij} = \frac{x_{ij}}{\max x_{ij}} \tag{3.1}$$

3. Calculation of the weighted normalized decision matrix.

The weighted normalized value  $v_{ij}$  is calculated in the following way:

$$v_{ij} = w_j n_{ij}$$
 for i=1,...,m; j=1,...,n (3.2)

where  $w_j$  is the weight of the j-th criterion,  $\sum_{j=1}^{n} w_j = 1$ .

4. Determine the positive ideal and negative ideal solutions.

Identify the positive ideal alternative (extreme performance on each criterion) and identify the negative ideal alternative (reverse extreme performance on each criterion). Positive ideal solution A<sup>+</sup> is calculated as:

$$A^{+} = (v_{1}^{+}, v_{2}^{+}, \dots, v_{n}^{+}) = \left( \binom{\max v_{ij} | j \in I}{i}, \binom{\min v_{ij} | j \in J}{i} \right)$$
(3.3)

Negative ideal solution A<sup>-</sup> is calculated as:

$$A^{-} = (v_{1}^{-}, v_{2}^{-}, \dots, v_{n}^{-}) = \left( \begin{pmatrix} \min v_{ij} | j \in I \\ i \end{pmatrix}, \begin{pmatrix} \max v_{ij} | j \in J \\ i \end{pmatrix} \right)$$
(3.4)  
where I is associated with benefit criteria and J with the loss criteria, i = 1, ..., m; j = 1, ..., n.

- 5. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

The separation of each alternative from the positive ideal solution is calculated as

$$d_i^+ = \left(\sum_{j=1}^n (v_{ij} - v_j^+)^p\right)^{1/p}$$
, i=1,2,...,m (3.5)

The separation of each alternative from the negative ideal solution is calculated as

$$d_i^- = \left(\sum_{j=1}^n (v_{ij} - v_j^-)^p\right)^{1/p}, i=1,2,...,m$$
(3.6)

where  $p \ge 1$ .

If p = 2 the n-dimensional Euclidean metric is used for calculation

$$d_{i}^{+} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{j}^{+})^{2}}, i=1,2,...,m,$$

$$d_{i}^{-} = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_{j}^{-})^{2}}, i=1,2,...,m,$$
(3.7)
(3.8)

6. Calculate the relative closeness to the positive ideal solution. The relative closeness of the i-th alternative  $A_i$  with respect to  $A^+$  is defined as

$$R_{i} = \frac{d_{i}^{-}}{d_{i}^{-} + d_{i}^{+\prime}}$$
(3.9)

where  $0 \leq R_i \leq 1, i = 1, 2, \dots, m$ .

Rank the preference order or select the alternative closest to 1.
 A set of alternatives now can be ranked by the descending order of the value of R<sub>i</sub>.