1. Construction of the decision matrix and determination of the weight of criteria. The multi-criteria problem can be expressed in matrix format in the following way:

$$\begin{array}{c|cccc} & C_1 & C_2 & \dots & C_n \\ \hline A_1 & X_{11} & X_{12} & \dots & X_{1n} \\ A_2 & X_{21} & X_{22} & \dots & X_{2n} \\ \dots & \dots & \dots & \dots & \dots \\ A_m & X_{m1} & X_{m2} & \dots & X_{mn} \end{array}$$

where:

$A_1, A_2, \dots, A_m$ – alternative of internal insulation systems/materials

$C_1, C_2, \dots C_n$ - the criteria for which the alternative performance is measured

x - the value of alternative A_i with respect to the criterion C_j.

Criteria of the functions can be either benefit functions when more is better, e.g. wall surface temerpature, CO_2 reduction, reduction of energy and other costs, reduction of energy consumption or loss functions when less is better, e.g. total costs of internal insulation. The relative importance of each criterion is given by a set of weights which are normalized to sum to one.

Let X = (xij) be a decision matrix and W = [w1,w2 , ... ,wn] a weight vector, where xij $\in\Re$, wj $\in\Re$ and w1 + w2 + …+ wn = 1.

2. Calculation of the normalized decision matrix.

Various attribute dimensions are transformed into non-dimensional attributes. It allows comparisons across criteria. Since various criteria are measured in various units, the scores in the evaluation matrix X have to be normalised to one scale. The normalization of values is carried out by standardized formula and the normalized value n_{ij} is calculated as:

$$n_{ij} = \frac{x_{ij}}{\max x_{ij}} \qquad (3.1)$$

3. Calculation of the weighted normalized decision matrix.

The weighted normalized value v_{ij} is calculated in the following way:

$$v_{ij} = w_j n_{ij} \text{ for i=1,...,m; j=1,...,n} \qquad (3.2)$$

where w_j is the weight of the j-th criterion, $\sum_{j=1}^{n} w_j = 1$.

4. Determine the positive ideal and negative ideal solutions.

   Identify the positive ideal alternative (extreme performance on each criterion) and identify the negative ideal alternative (reverse extreme performance on each criterion). Positive ideal solution A^+ is calculated as:

$$A^+ = (v_1^+, v_2^+, \ldots, v_n^+) = \left(\left(\max_i v_{ij} | j \in I\right), \left(\min_i v_{ij} | j \in J\right)\right) \qquad (3.3)$$

   Negative ideal solution A^- is calculated as:

$$A^- = (v_1^-, v_2^-, \ldots, v_n^-) = \left(\left(\min_i v_{ij} | j \in I\right), \left(\max_i v_{ij} | j \in J\right)\right) \qquad (3.4)$$

   where I is associated with benefit criteria and J with the loss criteria, i = 1, ... , m; j = 1, ... , n.

5. Calculate the separation measures from the positive ideal solution and the negative ideal solution.

   The separation of each alternative from the positive ideal solution is calculated as

$$d_i^+ = \left(\sum_{j=1}^{n} (v_{ij} - v_j^+)^p\right)^{1/p}, \text{ i=1,2,...,m} \qquad (3.5)$$

   The separation of each alternative from the negative ideal solution is calculated as

$$d_i^- = \left(\sum_{j=1}^{n} (v_{ij} - v_j^-)^p\right)^{1/p}, \text{ i=1,2,...,m} \qquad (3.6)$$

   where $p \geq 1$.
   If p = 2 the n-dimensional Euclidean metric is used for calculation

$$d_i^+ = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, \text{ i=1,2,...,m,} \qquad (3.7)$$

$$d_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \text{ i=1,2,...,m,} \qquad (3.8)$$

6. Calculate the relative closeness to the positive ideal solution. The relative closeness of the i-th alternative A_j with respect to A^+ is defined as

$$R_i = \frac{d_i^-}{d_i^- + d_i^+}, \qquad (3.9)$$

where $0 \leq R_i \leq 1$, i = 1,2, ... , m.

7. Rank the preference order or select the alternative closest to 1.
   A set of alternatives now can be ranked by the descending order of the value of R_i.